

Analytical Models

Analytical models are mathematical models that have a closed form solution, i.e. the solution to the equations used to describe changes in a system can be expressed as a mathematical analytic function.

For example,

A model of personal savings that assumes a fixed yearly growth rate, r , in savings (S) implies that time rate of change in saving $d(S)/dt$ is given by,

$$d(S)/dt = r(S) \text{ eqn. 1}$$

(This example is also used to describe [numerical models](#) so that numerical and analytical models can be compared and contrasted more easily).

The analytical solution to this differential equation is

$$S = S_0 \text{EXP}(r t) \text{ eqn. 2}$$

Where S_0 is the initial savings, t is the time, and $\text{EXP}(x)$ is Euler's number, e , raised to the power x . This equation is the analytical model of personal savings with fixed growth rate.

Are analytical models superior to numerical models? This may or may not be the case for introductory geoscience students. Some argue that analytical models are more aesthetically pleasing since an inspection of the mathematical function can give information about the system's behavior without the need for graphing or generating a table of values. This argument assumes that the person looking at the model has a command of mathematics, which may not be true for some introductory geoscience students.

Although the solution to the above simple system is fairly transparent, analytical solutions to equations describing more complex systems can often become fairly complicated. However, for those comfortable with mathematics an analytical solution does provide a concise preview of a model's behavior that is not as readily available with a numerical solution. Also implicit in the argument of an analytical model's superiority to numerical models is that graphing is tedious. This may have been the case 30 years ago but is certainly not true now. Regardless of whether one obtains an analytical solution or a numerical solution to a mathematical model, graphs showing the system's behavior over time and its sensitivity to variations in key model parameters are

essential for student understanding. One disadvantage of analytical solutions is that they are often very mathematically challenging to obtain.

Other resource

[Limited and Unlimited Growth](#) ([This site may be offline.](#)) This site discusses exponential growth and limited growth. It has an interesting JAVA type interactive applet at the bottom which is likely driven by the analytical solution to the logistic equation.

time (years)